The Relationship between Opportunities to Learn Algebra and Students' Algebra Achievement: A Comparative Study

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Abstract

The article provides a report on the comparison of Opportunities to Learn algebra and eighth grade students' algebra achievement in three countries: Botswana, Singapore and the United States. The study used student and teacher data from the Trends in International Mathematics and Science Study (2011 and 2015). By using a multilevel regression analysis, the study presents the models within the three countries of the relationships between Opportunities to Learn algebra and eighth-grade students' algebra achievement of the recent TIMSS cycle. The findings indicate that the Opportunities to Learn algebra are context specific and align with the expectations of the curriculum guides from the governing bodies within the countries. Also, the study shows that students who learned particular algebra topics in the early grades had significantly higher algebra scores in the TIMSS 2015 assessment. In particular, students who had been taught properties of functions and simplifying and evaluating algebraic expressions before eighth grade in the United States had significantly higher scores. Also, Singaporean students who had been taught simple linear equations and inequalities and simultaneous equations had higher algebra scores. Implications for policy and research are discussed.

Keywords

Curriculum, algebra achievement, achievement, Trends in International Mathematics and Science Study (TIMSS).

Introduction

The technological advancement in the 21st century will require greater numbers of more graduates with fluency in Science Technology Engineering and Mathematics related fields (Graham, Frederick, Byars-Winston, Hunter, & Handelsman, 2013). Notably, computer technology expertise, advanced sciences, economic and mathematics-related fields require graduates with fluency in the use of symbolic notation as a tool (Nofre, Priestley, & Alberts, 2014). Symbolic notation in mathematics is taught in the algebra strand of mathematics in middle school and is introduced by some teachers in the early elementary school years (Blanton & Kaput, 2005; MacGregor & Price, 2003). In the United States, Moses and team (1989) introduced the *Algebra Project* with an aim of making algebra available to all students in Cambridge, Massachusetts. They argued that access to algebra would provide

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students with a smoother transition to high school mathematics, and in turn provide a gateway to college entry. Therefore, success in learning algebra in elementary school may be viewed as a civil right for all students because of its importance in learning high school mathematics, college entry, and its application to various fields in the present technological age.

The teaching and learning of algebra is a challenge for teachers and students (Kierian, 2007; Rakes, Valentine, McGatha, & Ronau, 2010). Across nations, content guidelines and expectations differ on grade levels for teaching algebra and the depth of coverage (e.g., Chin et al., 2012; Common Core State Standards Initiative [CCSSI] 2010; Kesamang, Mudongo, Keatimilwe & Botaane, 2012). Therefore, a comparative analysis of the opportunities to learn algebra before high school, and relating these learning opportunities to students' algebra mastery is a step towards demystifying the learning of this component of mathematics. A cross-national analysis provides a broad scope for considering the possibilities of the influence of the Opportunities to Learn the algebra content on students' mathematics achievement.

This study drew on the Trends in International Mathematics and Science Study (TIMSS) 2011 and 2015 data to examine the changes in the Opportunities to Learn (OTL) algebra, and eighth-grade students' algebra achievement in three countries with markedly different TIMSS achievement scores: Botswana, Singapore, and the United States. Further, the research focuses on the influence of the recent OTL algebra on students' algebra achievement across the three countries. The questions guiding the study: (a) How did the Opportunities to Learn algebra and students' mathematics achievement differ between three countries in 2011 and 2015? (b) What was the intended and implemented algebra curriculum in elementary and lower secondary school in 2011 in the three countries? (c) What is the relationship between Opportunities to Learn algebra and eighth-grade students' achievement in algebra across the three countries in 2015 while controlling for students' background and gender?

The selection of each country in three continents considered: the language of instruction, previous TIMSS achievement scores, and curricular differences. These three countries use English as the language of instruction. Similar to having a coherent and rigorous curriculum like other higher-achieving nations, Singapore is unique in Asia for having classroom instruction conducted in English, and the curriculum materials provided in English (Wang-Iverson, Myers, & Lim, 2010). Botswana is one of the two Sub-Saharan African countries that took part in the TIMSS-2011 and 2015 that uses English as the language of instruction. South Africa, which is the other African country that participated in TIMSS in the two cycles was not selected because the language of instruction is either English or Afrikaans. For this reason, Botswana was preferred because of single English language policy for instruction. The United States is unique as a country because it consists of several states that all use English as the language of instruction and therefore provides variation in education policies that may exist among the different states.

The TIMSS achievement scores were also used to select the countries for inclusion in the study. Singapore has consistently posted a high score on international assessments at the eighth-grade level (Mullis, Martin, & Foy, 2008; Organization for Economic Cooperation and Development [OECD], 2010). Therefore, information from a high-achieving nation serves as a benchmark for lower achieving nations, such as Botswana. Botswana is one of the African countries that has shown steady economic and political stability and is unique in providing educational resources to the entire school-aged population. However, with all these positive attributes, it still posted a low score in both TIMSS math assessments. Curricular differences are also worth noting as a factor for selection of countries. Singapore and Botswana all use a national curriculum, whereas in United States the state education and the local school district agencies decide on the curriculum and accountability measures that are aligned with the standards shared across the United States. Beginning in 2010 the United States began to transition to a national curriculum and by 2015, 43 states, four US territories, and the Department of Defense Education had adopted the common core state standards (National Governors Association of Best Practices & Council of Chief State School Officers [NGA & CCSSO], 2010; Malley, Neidorf, Arora, Kroeger & American Institutes of Research, 2016)

International comparisons may provide vital information for policy and practice even though countries may have different cultural patterns, beliefs, educational structures, curriculum, and expectations (Desimone, Smith, Baker, & Ueno, 2005; Schmidt et al., 2001). This study compared the in-country relationships in three countries rather than making comparisons of the relationships across the selected countries, i.e., in each selected country the Opportunities to learn algebra were analyzed as cases and the comparisons of the relationships between these Opportunities to learn and algebra achievement within the countries then compared. This large-scale comparative study across varying contexts, both within and between the three countries, allows for a broader consideration of influencing factors on student achievement and provides a more extensive generalization of findings. The similarities and differences of factors related to student achievement across contexts, which are not usually apparent, can be illuminated using such a study.

Theoretical Perspective

The theory guiding this study draws on the Opportunity to Learn (OTL) framework. The literature provided is organized by first discussing the OTL framework. OTL is also elaborated in regard to the intended curriculum and implemented curriculum. Following is a summary of studies on algebra learning as well as those on the relationships between OTL algebra and student achievement. Included in the literature are discussions on the connections between the documented studies and the presented research.

Opportunity to Learn

Carroll (1963) introduced Opportunities to Learn in the proposed dimensions of the model of learning. According to Carroll, the OTL dimension falls within the external conditions that can be adjusted to improve learning. OTL was defined as “the amount of time allowed for learning” (Carroll, 1963, p. 26). The expanded definitions of OTL are the intended time allocated to learn or teach a topic in the national curriculum guides or reported by teachers. It is also the proportion of teachers in a country covering the topic (Schmidt et al., 2001) or the relative emphasis on a specific topic (Floden, 2002). Floden (2002) elaborated that the time teachers plan to spend teaching a topic is another measure of OTL. In addition, the time teachers actually spent teaching a topic or the time a student is present when learning a topic is measured as an OTL. OTL is also the degree to which a student engages in specific instructional activities (Floden, 2002). Schmidt and colleagues (2001) and Floden (2002) categorized OTL into the *intended* or the *implemented* curriculum. The intended curriculum includes the content standards used to guide instruction, whereas the implemented curriculum represents the percentage of instructional time spent or teachers’ reports on coverage of a topic (Schmidt et al., 2001). Similarly, Floden (2002) added that the intended curriculum includes the topics scheduled to be taught in a school year or the time a teacher sets aside to teach the topic. In addition to Schmidt and colleagues (2001) summary of the implemented curriculum as an OTL, Floden (2002) included the count of the textbook pages on the topic, the teacher's report on the emphasis of the topic in relation to other topics (Floden, 2002). Floden (2002) stated that OTL is also the academic learning time which is the time that students are engaged in learning or the estimated time students complete a given task successfully or paying attention.

Other definitions of OTL include approaches to teaching experience (Blömeke & Delaney, 2014; Schmidt, Blömeke, et al., 2011; Schmidt, Cogan, & Houang, 2011), access to qualified teachers, resources for learning, and the goals set by instructors (Oakes, 1990). OTL is a focal area of study in international comparisons (Floden, 2002) and was found to be positively related to students' mathematics achievement (Floden, 2002; Schmidt et al., 2001; Törnroos, 2005). This study draws from Floden (2002) and Schmidt and team's (2001) OTL that is categorized as the intended and implemented curriculum. In particular, the intended curriculum for this study is the schedule or time for an algebra topic to be taught in a school year. The implemented curriculum is the teachers' reports on the coverage of an algebra topic. OTL might explain the variations in student mathematics achievement across nations that can inform policy decisions on curriculum preparation and promising opportunities that make significant improvements in students' learning.

Algebra Learning

Scholars have found that teaching algebra in the early elementary grades (Grades 1-5) is possible and critical to students' success in learning more advanced algebra in secondary school. Knuth, Stephens, Blanton, and Gardiner (2016) suggested the infusion of algebra in mathematics learning in the earlier grades. In particular, the use of variables, introducing the equal sign as relational instead of an operation, and functional relationships were taught with notable success among third-fifth grade students (Knuth et al., 2016). Early studies on algebra learning document positive outcomes with the introduction of functional thinking in lower elementary (e.g., Martinez & Barbara, 2006; Tanışlı, 2011; Warren, Cooper, & Lamb, 2006). Notably, the nature of selected mathematics tasks (Bjuland, 2012; Gardiner & Sawrey, 2016) and approaches to teaching algebra (Chan, 2015; Whitin & Whitin, 2014) enabled first and fourth grade students to engage in functional thinking and algebraic reasoning. Similarly, Blanton, Stephens, Knuth, Gardiner, Isler and Kim (2015) reported that third grade students who were engaged in an early algebra intervention were able to correctly interpret the equal sign as a relational symbol, represent unknown quantities using variable notation, use a structural approach to solve selected tasks, and represent relationships of variables using covariational reasoning. According to Blanton and team (2015) the algebraic reasoning the third graders exhibited were critical benchmarks for the development of algebraic thinking that often proved to be a challenge for students in the middle grades and beyond.

One of the factors related to the learning of algebra is the opportunity to learn**.** Therefore, a focus on the grade level when students learned or were taught algebra tasks could explain the advantages of their success at the eighth-grade level. This study adapted the conceptualization of OTL used by Floden (2002) and Schmidt et al. (2001), which is the percentage of instructional time spent on the topic and the curriculum offered. Specifically, this study compared the OTL algebra content of mathematics in two years (2011 and 2015) in the three countries.

OTL Algebra in the Three Countries

The curriculum for teaching algebra varies across the selected countries. In Botswana, the mathematics curriculum in algebra is concentrated in the upper primary (Grades 5-7) and lower secondary (Grades 8-10). In upper primary suggested algebra topics include using algebra to communicate numbers, number patterns and arithmetic sequences, solving and simplifying linear expressions (Masole, Gabalebatse, Guga, Pharithi, 2016; Botswana Examination Council, 2016). In lower secondary the algebra curriculum includes simplification of algebraic expressions, expansion and factorization, evaluation and formulae for numeric values, and formation and solving linear and systems of equations using real-life situations, patterns, and sequences (Barungwi, Cele, Mudongo, Morake, & Hlomani, 2008; Masole, et al., 2016; Ministry of Education Botswana, 2010).

In Singapore, algebra content in the primary (Grades 1-6) includes algebraic expressions with one variable and lower secondary (Grades 7-8) includes quadratic equations, linear inequalities, simultaneous equations, linear equations, algebraic expressions and formulae, rate and speed, algebraic manipulation, and *set* language and notation, functions and graphs (Chee et al., 2016; Quek et al., 2008; Ministry of Education Singapore, 2007). At eighth grade, a substantial section of the mathematics curriculum is algebra (Kaur, 2008).

However, in the United States, the curriculum guides vary across the different states and school districts (Keene, 2008; Malley, et al., 2016). In algebra, the curriculum topics in middle school (Grades 6-8) include numeric values, algebraic patterns, geometric patterns or sequences, powers of expressions containing variables (National Council of Teachers of Mathematics [NCTM], 2000). Other topics include evaluating expressions for a given numeric value, simplification and comparison of algebraic expressions, using expressions to model situations, simple linear equations, simultaneous equations, and multiple representations of functions, function properties, radicals and integer exponents (Keene, 2008; Malley, et al., 2016). While the United States in America claimed to adopt the Common Core State Standards of Mathematics in 2011(NGA & CCSSO, 2010), the instruction provided in this study followed NCTM standards.

Figure 1 summarizes the hypothesized relationships of this study (including the background variables: gender and academic resources (background). Students' gender and academic resources (background) are used as control variables because they are factors that have been found to influence student academic achievement. The students' background variables are a Socio-economic measure, a term introduced by Carnoy (2015), which includes parental level of education and the number of books in the students' home. I hypothesize that those students who have had the OTL various content areas of algebra in before and at eighth grade have higher algebra achievement, controlling for their background and gender.

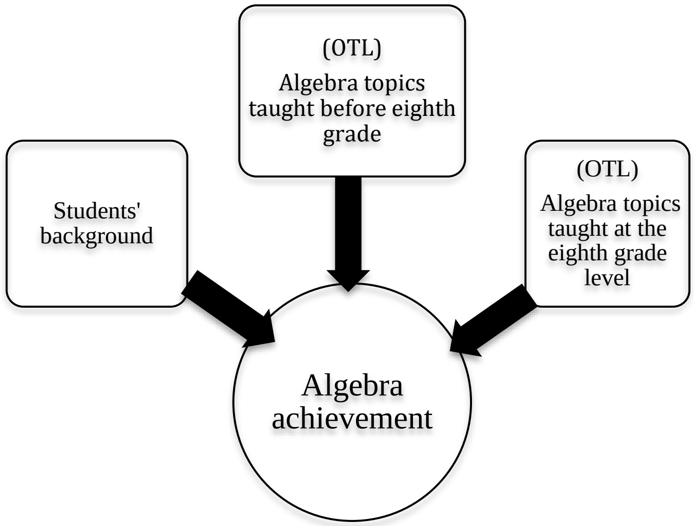

Figure 1. The hypothesized relationships between the OTL algebra and eighth grade students' algebra achievement.

Method

Data and Sampling

The study used data from TIMSS 2011 and 2015. TIMSS 2011 and 2015 are the fifth and sixth cycles of the TIMSS comparative assessment of mathematics and science around in the world, respectively. These studies are cross-sectional surveys of information about the teaching and learning of mathematics cross-nationally in the fourth and eighth grades of school. The fourth grade was considered as the students who have been in school for four years beginning from first grade with a mean age of at least 9.5 years. Similarly, students sampled for the eighth-grade study have been in school for the last eight years and mean age of at least 13.5 years (Laroche, Joncas & Foy, 2016). The data were collected through a disproportionate sampling procedure in which whole classes were selected after dividing the target population into strata (Olson et al., 2008). The schools were sampled in each of the strata in the first stage and then the classes within the schools in the second stage. Stratification improves efficiency, makes estimates more reliable, and allows for differences in sampling based on the population years (Laroche, Joncas & Foy, 2016). The variables used for stratifying the samples were agreed upon by consultation between the National Research Centers of the participating countries and Statistics Canada (Mullis Martin, Foy & Arora (2012); Mullis, Martin, Foy & Hooper (2016)

TIMSS aims to get a 100% participation rate for the sampled schools and classes. The sampling team set the required participation rates at 85% minimum for the schools and 95% for the classrooms, or a combined participation rate of 75% years (Laroche, Joncas & Foy, 2016). For each of the sampled schools, two schools were identified as replacement schools in the specified stratum. In order to cater for the differing sizes of the schools and classes and non-participation, unique sampling weights were introduced (Laroche, Joncas & Foy, 2016). The sampling weights were calculated for each grade and study and were taken into consideration during any TIMSS analysis.

The TIMSS data was collected from students, teachers, principals, and education representatives in the participating countries.

Background information was collected from the students and their teachers. In addition, the students and their teachers responded to questions about the learning and teaching of mathematics. Information about the curriculum was obtained from education representatives in each of the participating countries. The principals in the participating countries provided information about their schools. In sum, 45 countries and 14 benchmarking entities participated in 2011 and 57 countries and 7 benchmarking entities in 2015. The information gathered from the data informs policy and practice towards the improvement in teaching and learning of mathematics and science.

The study focused on three countries, Botswana (n=5400 in 2011; n=5964 in 2015), Singapore (n=5927in 2011; n=6116 in 2015), and the United States (n=10,477 in 2011; n=10,491 in 2015) that participated in the TIMSS -2011 and 2015. The International Association for the Evaluation of Educational Achievement (IEA) had national research coordinators from each of the participating countries conducting the survey. The target population was students, and their teachers, who were classified by the International Standard of Classification of Education to be in eighth grade or its equivalent. In Botswana, the ninth-grade students responded to the eighth-grade survey but in Singapore and the United States the eighth-grade students were the respondents in the study. The study relied on the students' and teachers' self-reports on the opportunities to learn specific algebra content and the amount of time devoted to teaching algebra and the students' algebra achievement. Table 1 presents a description of the variables used in the study.

Analysis

The first two research questions required descriptive information of the OTL algebra and eighth-grade student mathematics achievement information in the three countries.

Research question one: "How did the Opportunities to Learn algebra and students' mathematics achievement differ across the three countries in 2011 and 2015?" This research question was answered using means, standard deviations that illustrate the variations in OTL and students' mathematics achievement in the three countries, and a one-way ANOVA to test for differences in the means. The analysis included mean differences by country and a combination of data from 2011 and 2015 with mean difference tests conducted where appropriate. These analyses were conducted using the IEA International Database Analyzer (IDB). The IDB analyzer was used to merge the files for the three countries and for descriptive analysis of the merged data. This software is a *plugin* used in the SPSS[1] (IBM Corporation, 2012) platform that considers the complex sampling in selecting the appropriate weight and correcting for the standard error.

Descriptive information on teachers' reports about when they intended to teach specific algebra content as well as the percentages of teachers whose reports included when they taught specific algebra content was used for the analysis of the second research question: "What was the intended and implemented algebra curriculum in elementary and lower secondary school in 2015?" Data was collected from respondents in the respective countries providing information on the intended curriculum and data available from the TIMSS database.

Research question three, "What is the relationship between Opportunities to Learn algebra and students' achievement in algebra across the three countries?" was examined using a multi-level regression analysis. This analysis

Table 1
Variables used in the study

Main Variable	Variables	Type	Description
Opportunities to Learn algebra (Intended)	Intended grade algebra topics should be taught	Ordinal	From the curriculum questionnaire the respondent selects when topic is intended to be taught. The respondent in this case is either the curriculum director or the principals in the selected schools.
Opportunities to Learn algebra (Implemented)	Algebra topics taught	Ordinal	Teacher reports on whether topics were taught before or at eighth-grade.
	Time spent teaching algebra and other topics	Ordinal	The teacher reports what percentage of time they will have spent teaching algebra and other topics by the end of the school year†.
Student achievement	Mathematics achievement	Continuous	Students' achievement in mathematics content of algebra, data and chance, geometry and numbers.
Gender	Students' gender	Dichotomous	Students gender used as a control variable
Socio-economic Status	Number of books in the home	Continuous	A proxy variable selected to be used as a measure of the students' socio-economic status.
	Highest parental level of education		Mother and fathers' level of education given using the ISCED classification

† The categories answered are: mostly taught this year, taught before this year, and the topics not yet taught or just introduced. In this study the responses are recoded such that mostly taught this year and taught before this year were the percentages considered.

technique is useful because it considers the cluster sampling. In particular, the sampling design, weighting, and the mathematics achievement given in the form of five plausible variables (Foy & Olson, 2008; Rutkowski, Gonzalez, Joncas, & von Davier, 2010) was considered using HLM[2] software (Raudenbush & Bryk, 2002).

The student level variables were set at level 1 and the teacher variables at level 2.

Level 1 (Student level)

$Algebra\ achievement_{ij} = \beta_{0j} + \beta_{1j}*(Gender_{ij}) + \beta_{1j}*(Number\ of\ books\ in\ the\ home_{ij}) + \beta_{1j}*(Parents\ education\ level_{ij}) + r_{ij}$

β_{0j} is average algebra score in institution *j*. The variables represent the group- mean centered independent variables that include the covariates at the student level. The beta coefficients represent the slope or the corresponding change in algebra score for every unit change in the covariates. The error of the prediction of the equation is represented by r_{ij}.

Level 2 (Teacher level)

Model 1: $\beta_{0j} = \gamma_{00} + \gamma_{01}*(OTL\ algebra\ before\ eighth\ grade_{2\text{-}6j}) + u_{0j}$

Model 2: $\beta_{0j} = \gamma_{00} + \gamma_{01}*(OTL\ algebra\ at\ eighth\ grade_{6\text{-}10j}) + u_{0j}$

The OTL variables are the grand mean-centered independent variables at the teacher level. γ_{00} is the grand mean algebra score of the sample in the country. u_{0j} is the random error for deviation of group intercept with the overall intercept. This analysis was run separately for the two cycles.

Results

Mathematics Achievement in 2011 and 2015

In the TIMSS study, each student answered a few items from the more extensive pool for mathematics assessment. The limited responses of the students were used to generate an ability distribution for each of the students, from which five plausible values calculated (Rutkowski et al., 2010). The analysis provided the combined *plausible values* for each student found through the use of the IDB analyzer. Figure 2 presents the graph of the differences in mathematics achievement scores in the content areas over the two years.

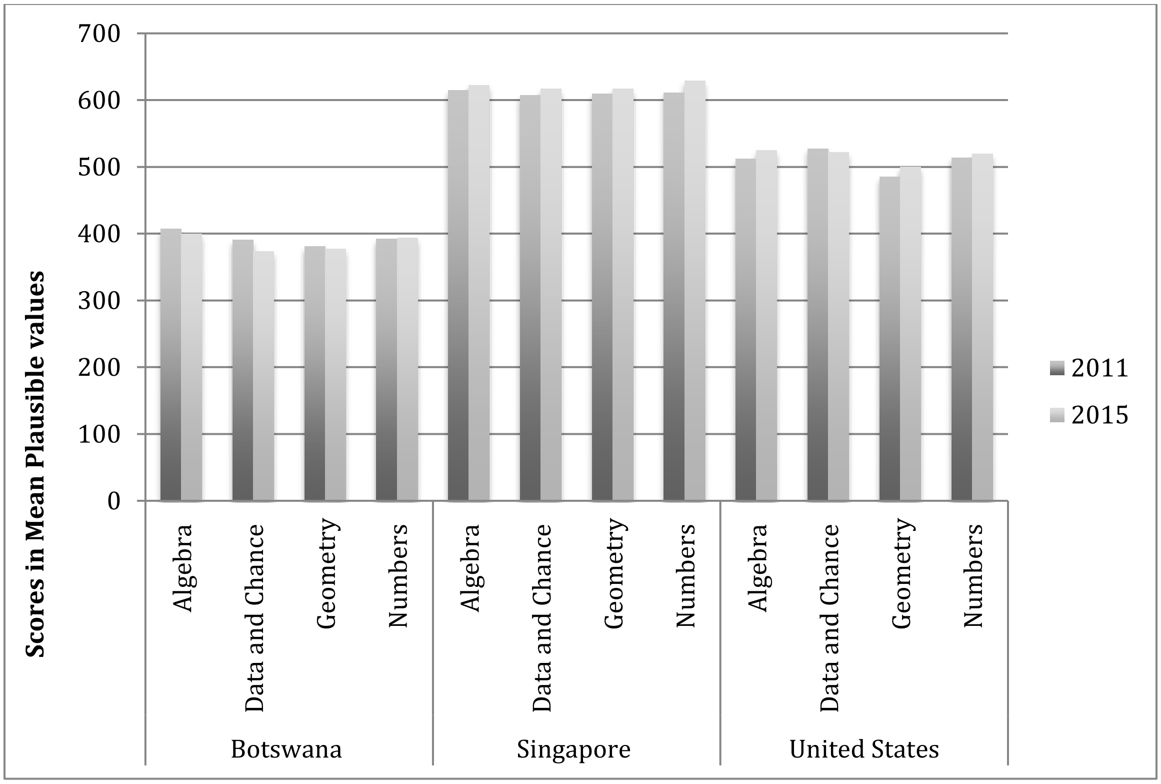

Figure 2. Mathematics achievement scores in 2011 and 2015 in the four topical areas (Mean plausible values).

A descriptive analysis of the scores by average math scores between the two TIMSS cycles, content areas, and between countries indicated that there were differences in the students' scores worth mentioning. In 2015 the average students' mathematics scores was higher((M=512.19, SD 115.34) than in 2011(M=507.62, SD =108.58), t(44, 103) =-4.284, p<0.001. That is, a combination of the math scores in 2011 and 2015 were significantly different. A further analysis of each content area for the three countries combined also indicated there was some significant differences. In particular , the combined mathematics scores in *numbers* were higher in 2015 (M=517.05, SD 118.79) than 2011 (M=509.19, SD 114.08), t(44,103)=-7.085, p<.001). A comparison of the mean mathematics scores for the three countries revealed that Botswana had the lowest mean grade when compared to the other two countries, whereas Singapore had the highest scores in all the mathematics topics among the three countries in both cycles. Botswana had a significant lower average score in all mathematics content areas in 2015 when compared to the United States (p<.001) and Singapore (p<.001).

Algebra was the content area in which the eighth-grade students in Botswana and Singapore had the highest mean scores in 2011(406.81 and 614.47 respectively) in all content areas within the countries. In 2015 the eighth-grade students in Botswana still had the highest mean content score in *Algebra* (399.82), but the eighth-grade Singaporean students had their highest mean content score in *Numbers* (628.95). In the United States, the highest mean score was in *Data and Chance* in 2011(527.86) to *Algebra* in 2015 (524.86). Finally, the students in the United States and Singapore maintained the lowest mean grade in geometry content. Notably, in Botswana, the eighth-grade students lowest mean score shifted from *Geometry* in 2011 (380.68) to *Data and Chance* in 2015 (373.56). Although *algebra* was the content area in which students performed highest in Botswana, the general performance in algebra was significantly lower (p<.001), when compared to the other two countries.

OTL (Intended Curriculum)

The second research question was an inquiry of the intended and implemented curricula in 2011. In this section, the report provides a descriptive summary of the intended curriculum as analyzed from the TIMSS data base, collected through the curriculum questionnaire in which the principals or the curriculum directors responded. The intended curricula in Botswana, Singapore, and the United States indicate the algebra topics were taught at, and introduced at, different grade levels. For instance, *simultaneous equations* should be taught at the eighth-grade level in all three countries. *Simplifying and evaluating expressions* was to be taught from grade six onwards in Botswana and Singapore, but to be taught only at the sixth-grade level in the United States and some sections of the topic to be taught at the fourth-grade level in Botswana. Notably, the Singaporean curriculum expectations emphasized that *patterns and sequences* were to be taught all eight years from first grade, but to be taught from third grade in Botswana and fourth grade in the United States. The other algebra topics were taught in the upper elementary grades and taught at subsequent years, but the grade levels at which the topics were first taught differed. A summary table of the topics and the grade levels when topics were to be taught is summarized in Table 2.

Table 2

Intended Algebra Curriculum Grades 1-8 in Botswana, Singapore, and the United States

Algebra Topics	**Country**	**Grades**							
		1	2	3	4	5	6	7	8
Simplifying and evaluating algebraic expressions	Botswana				X		X	X	X
	Singapore						X	X	X
	United States						X		
Simple linear equations and inequalities	Botswana					X	X	X	X
	Singapore							X	X
	United States						X	X	X
Simultaneous (two variables) equations	Botswana								X
	Singapore								X
	United States								X
Numeric, algebraic, and geometric patterns or sequences (extension, missing terms, generalization of patterns)	Botswana			X	X	X	X	X	X
	Singapore	X	X	X	X	X	X	X	X
	United States				X	X	X	X	X
Representation of functions as ordered pairs, tables, graphs, words, or equations	Botswana						X	X	X
	Singapore							X	X
	United States					X	X	X	X
Properties of functions (slopes, intercepts etc.)	Botswana								X
	Singapore							X	X
	United States						X	X	X

Implemented Curriculum

Algebra Topics Taught Before Eighth Grade

The percentage of teachers that reported teaching various algebra topics before eighth grade changed between 2011 and 2015. In all three countries, the proportion of the students' teachers who taught *simple linear equations and simplifying and evaluating algebraic expressions* before eighth grade increased markedly between the years 2011 to 2015. In particular, the percentage of teachers who taught *simple linear equations* increased from 13.79% to 18.85% in the United States, 31.62% to 40.39% in Singapore, and 16.09% to 19.49% in Botswana. Similarly, the percentage of teachers who reported that they taught *simplifying and evaluating algebraic expressions* increased from 18.75% to 29.87% in the United States, and 28.69% to 48.41% in Singapore. Also, there was an increase in the number of Singaporean (13.88% to 24.7%) and Botswana (15.37% to 18.72%) teachers who reported they taught *multiple representations of functions* across the two cycles. However, there was a decrease in the percentage of teachers who taught *patterns and sequences* in the United States (36.08% to 21.43%) and Botswana (50.46% to 40.38%). Finally, there was an increase in the percentage of teachers who reported teaching *simple linear equations* from 2011 to 2015 in all three countries. That is, there was an increase from 13.79% to 18.85% in the United States, 31.62% to 40.39% in Singapore, and 16.09% to 19.46% in Botswana.

Algebra topics taught at the eighth-grade level

Most of the algebra content was taught in the eighth grade in the three countries but with lower percentages of teachers teaching particular topics in the three countries in 2015. Specifically, a decreased percentage of teachers across the three nations reported teaching *simplifying and evaluating algebraic expressions* at the eighth-grade level (76.91% to 68.75% in the United States, 71.31% to 51. 59 in Singapore, and 52.16% to 46.16 % in Botswana). Notably, there was a drop in the percentage of teachers that reported having taught *simple linear equations and inequalities* (55.22to 40.14%), *numeric, algebraic, and geometric patterns* (44.04% to 32.18%), and *simultaneous equations* (32.77% to 16.59%) at the eighth-grade level in Botswana. However, in the United States, there was an increase in the percentage of teachers that reported having taught *representations of functions* (80.74% to 88.42%) and *simultaneous equations* (58.45% to 79.73%) at the eighth-grade level. Finally, the highest percentage of teachers that reported that they taught *properties of functions* at the eighth-grade level (89.53%) was in the United States, whereas Botswana had the lowest percentage (30.88%). These results indicate the OTL the different algebra topics that represent the implemented curriculum as reported by the students' teachers differed in the three countries. Also, most of the algebra curriculum seemed to have been implemented at the eighth-grade level as shown by the higher percentage of teachers who reported when they taught the algebra topics. Figures 3-5 provide summaries of the OTL various algebra topics before and at the eighth-grade level.

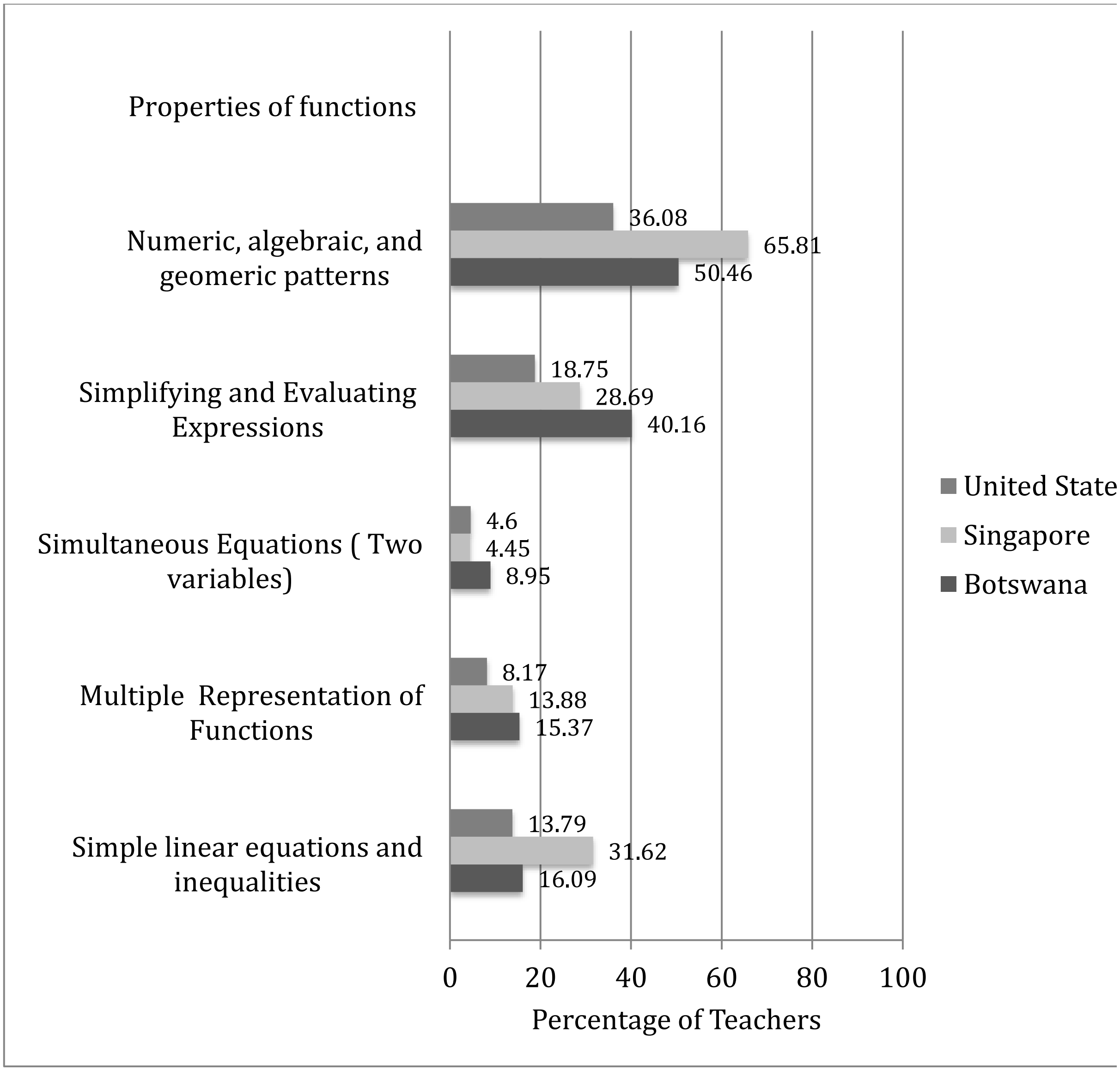

Figure 3. Percentage of teachers who reported having taught specific algebra topics before eighth grade in 2011.

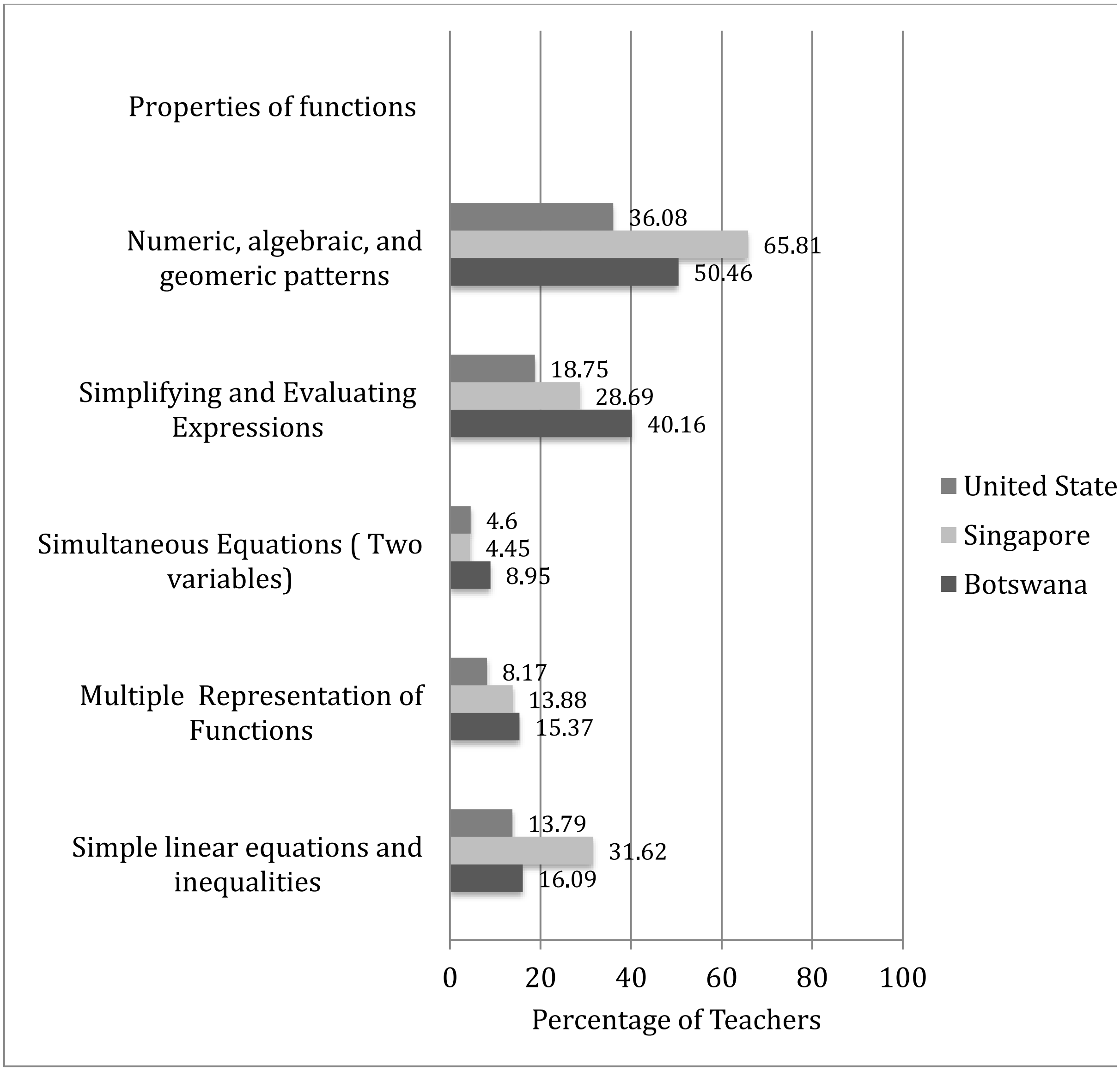

Figure 4. Percentage of teachers who reported having taught specific algebra topics before eighth grade in 2015.

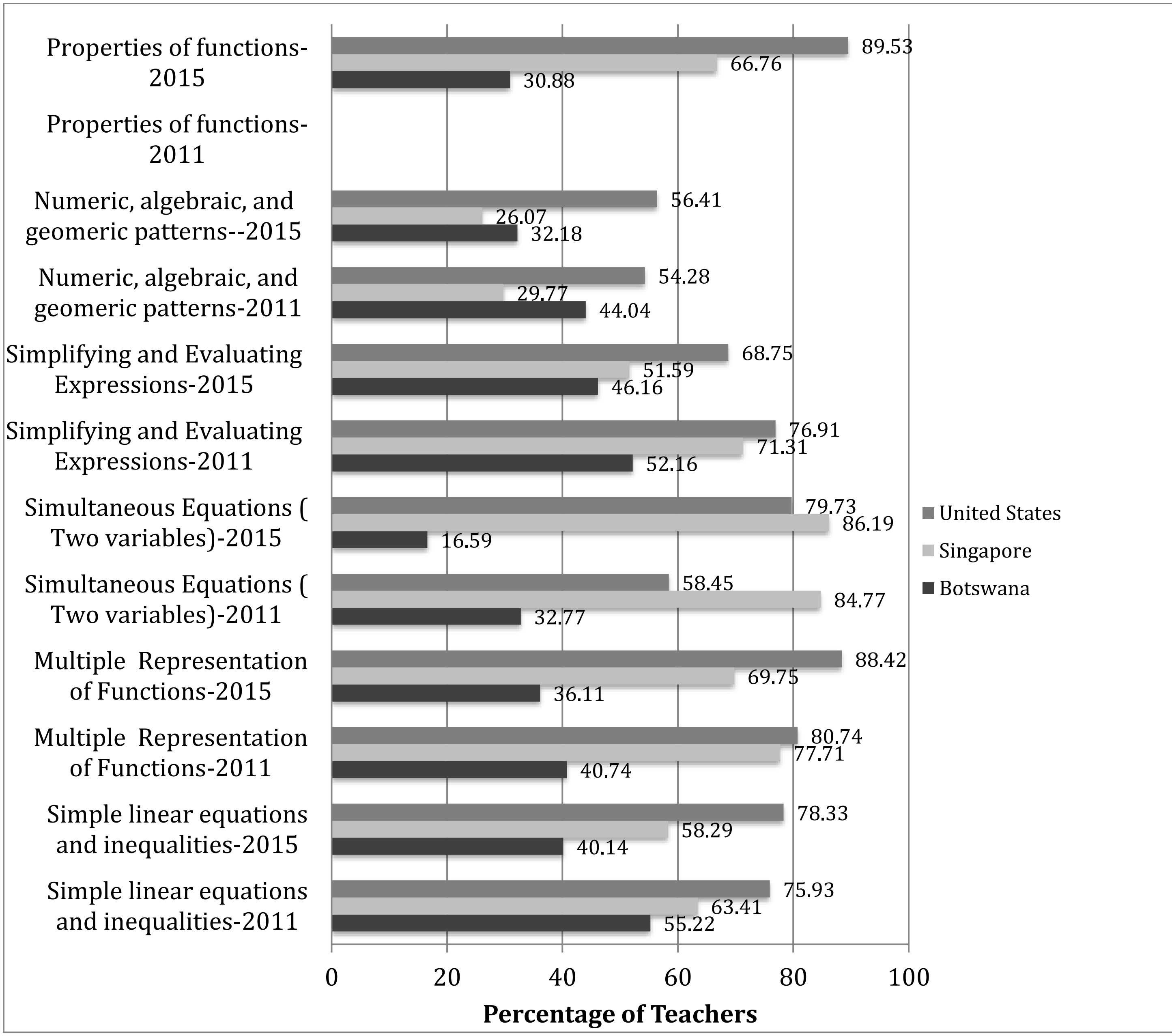

Figure 5. Percentage of teachers who reported teaching specific algebra topics at the eighth-grade level in 2011 and 2015.

Relationships between OTL Algebra and Students' Algebra Achievement in 2015?

The 2015 data contributes information on the most recent relationships between OTL algebra and algebra achievement in the selected countries. A multi-level model of the relationships between the OTL algebra and students' algebra achievement in 2015 is presented in Tables 4 and 5. Table 4 is the unconditional model of the algebra achievement, and Table 5 provides the multi-level models of the relationships between the OTL algebra and algebra achievement within the three selected countries. The tables include the values of the beta coefficients and the standard errors in parenthesis. The Intra-class correlations (ICC) is the ratio of the between-class variation and the total variance in algebra achievement ($ICC = \tau_{00} / [\tau_{00} + \sigma^2]$). The ICC values for algebra achievement in the three countries are above 0.1(see Table 4) and therefore it is permissible to use a multi-level model (Raykov & Marcoulides, 2012). The ICC values provide information about the variation of algebra achievement scores between the students' teachers.

Table 4
Unconditional models of Eighth Grade Students' Algebra achievement in 2015

Variables	Botswana	Singapore	United States
Intercept	401.39***	619.95***	523.12***
	(3.12)	(4.43)	(3.35)
Variance Components			
Variance between students' teachers	1023.34	5899.96	3859.75
Variance between students	5116.87	1948.86	3594.20
ICC	0.17	0.75	0.52

***p<.001

Table 5

Relationships between OTL algebra and Eighth Grade Students' Achievement in Algebra in 2015

Variables	Botswana		Singapore		United States	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
	399.38***	399.60***	619.31***	619.62***	524.92***	524.38***
Intercept	(3.28)	(3.25)	(3.89)	(3.73)	(3.25)	(3.20)
Female	23.08***	23.08***	-.53	-.53	3.79**	3.79**
	(3.58)	(3.81)	(2.99)	(2.99)	(1.85)	(1.85)
Number of books in the home	2.42	2.42	-.25	-.25	8.92***	8.93***
	(2.07)	(2.07)	(1.23)	(1.22)	(.89	(.89)
Fathers educational level	4.29***	4.29***	.51	.51	1.06	1.06
	(.85)	(.93)	(.32)	(.32)	(0.78)	(.78)
Taught before eighth grade						
Numeric, algebraic and geometric patterns	11.80		1.38		-14.69	
	(6.80)		(8.74)		(8.31)	
Simplifying and evaluating algebraic expressions	7.47		-7.67		18.18**	
	(7.03)		(8.92)		(8.38)	
Simple linear equations and inequalities	-4.30		63.41***		15.71	
	(11.40)		(9.84)		(10.02)	
	17.39		56.23**		-5.92	
Simultaneous equations	(16.33)		(17.96)		(18.52)	
	-1.82					
Representations of	(10.87)		22.24		-6.98	
functions			(12.79)		(16.09)	
	-10.88		2.92		54.64**	
Properties of functions	(13.41)		(12.68)		(20.97)	

Taught at eighth grade						
Numeric, algebraic and geometric patterns		-10.11		-3.07		15.81**
		(6.54)		(8.78)		(6.97)
Simplifying and evaluating algebraic expressions		8.01		-1.32		23.86**
		(5.61)		(8.33)		(7.50)
Simple linear equations and inequalities		-5.95		-50.32***		-13.47
		(5.55)		(9.10)		(8.74)
		16.60		74.45***		27.67**
Simultaneous equations		(8.86)		(10.14)		(9.96)
Representations of functions		8.39		-17.24		-.05
		(5.97)		(9.94)		(12.19)
		6.77		-13.08		-22.13
Properties of functions		(7.27)		(9.63)		(14.23)
Variance Components and fit index						
χ^2	909.89***	876.48***	14046.0***	12.702.99***	7561.04***	7534.21***

*p<.05, **p<.01, ***p<.001

The findings from the multi-level modeling of the relationship between the OTL algebra and students' algebra achievement indicate that particular relationships were context-specific, and the relational patterns differed across the three countries. The interpretation of the relationships is reported by first considering the associations between OTL algebra before the eighth-grade and students' algebra achievement. Next, is a discussion of the relationships between OTL algebra at eighth grade and students' algebra achievement. All these relational findings are controlled for gender and the students' academic resources(background) at home.

Teaching algebra topics before eighth grade

The findings indicate there were differing patterns in the relationships in all three countries between OTL algebra before eighth grade and the students' algebra achievement. Specifically, students in Singapore whose teachers reported they taught *simple linear equations and inequalities* before eighth grade had significantly higher scores (β=63.4, p<. 001) when compared to those who did not learn this algebraic topic or were taught the topic at the eighth-grade level. Further, the Singaporean students' whose teachers reported they were taught *simultaneous equations* before eighth grade had significantly higher scores (β=56.23, p<. 05) than those who had not been taught the topic or had this OTL the eighth-grade level.

In Botswana, students whose teachers reported that they had been taught *numeric, algebraic and geometric patterns, simplifying and evaluating algebraic expressions,* and *simultaneous equations* had higher scores than those who had not been taught the topic or were taught in the eighth grades. In contrast, students whose teachers reported they had taught *simple linear equations and inequalities* and *properties of functions* had lower scores when compared to those who had not been taught the topics or had been taught at the eighth-grade level. Notably, all the relationships between the OTL algebra before eighth grade and students' algebra achievements were not significant in Botswana.

In the United States students whose teachers reported having taught some algebra topics before eighth grade had significantly higher algebra scores. Mainly, students whose teachers reported having taught *simplifying and evaluating algebraic expressions* and *properties of functions* at the eighth-grade level had significantly higher scores (β=18.18, p <. 05 and β=54.64, p <. 05, respectively). These scores were compared to students who had either not been taught the topic or who experienced this OTL at the eighth-grade level. However, those students whose teachers reported they taught *numeric, algebraic, and linear patterns* and *simultaneous equations* before eighth grade had lower scores in the United States.

Teaching algebra topics at the eighth-grade level

Across the three countries, the pattern of relationships between OTL algebra topics and students' algebra achievement differed but were similar for two algebra topics. The relationships between OTL *simultaneous equations* and students' algebra achievement were positive in all three countries and significant in Singapore (β =74.45, p<. 001) and the United States (β =27.67, p<. 05). That is, students whose teachers reported that they had been taught these topics at the eighth-grade level had significantly higher algebra scores when compared to those who had not been taught these topics. In contrast, the students whose teachers reported that they had been taught *simple linear inequalities* at the eighth-grade level had significantly lower algebra scores when compared to those who had not been taught this topic or had been taught it at an earlier grade, for the Singaporean students (β=-50.32, p< .001). Notably, there was a significant positive relationship between OTL *numeric, algebraic and geometric patterns* and students' algebra achievement in the United States (β=15.81, p<. 05), but that relationship was negative in Botswana and Singapore. Finally, there was a significant positive relationship between OTL simplifying and evaluating algebraic expressions and students' algebra achievement in the United States (β=23.86, p<. 05). In other words, the students whose teachers reported that they had been taught about patterns at the eighth-grade level in the United States had significantly higher algebra scores when compared to those who had not been taught the topic at the eighth-grade level, whereas in the other two countries the students had significantly lower algebra scores. The other relationships between OTL representation of functions and properties of functions had differing patterns in the three countries and were not significant.

Discussion

The purpose of this study was to investigate the changes in the OTL algebra in three selected countries in 2011 and 2015. Additionally, the study also focused on the relationships between the OTL algebra topics and eighth-grade students' algebra achievement in the most recent TIMSS cycle, 2015. The investigations were done using the TIMSS (2011 and 2015) data. The findings from the study suggest that over the two cycles, there was an increase in the percentage of the eighth-grade students' teachers who reported that they had taught particular algebra topics before and in eighth grade. Further, across the three selected countries relationships between OTL various algebra topics and eighth-grade students' algebra achievement had differing patterns within the selected countries. Following is a discussion of the findings considering the country policies and earlier studies.

Opportunities to Learn Algebra

Botswana

There was a significant increase in the percentage of the eighth-grade students' teachers who reported that they had taught, *simple linear equations and inequalities* and *simplifying and evaluating expressions* before eighth grade in 2011 and in 2015. However, the proportion of teachers who reported they taught *numeric, algebraic, and geometric patterns* before eighth grade was significantly lower in 2015. A comparison of 2011 and 2015 data indicated that the percentage of teachers who taught the different algebra topics at the eighth-grade level dropped. In particular, there was a 6% drop in the percentage of students' teachers who reported they had taught *simplifying and evaluating expressions* at the eighth-grade, a 15.08% drop who reported they had taught simple linear equations, a 11. 86% drop for those who reported they had taught algebra topics on patterns, a 16.18% drop in those who reported they had taught simultaneous equations, and 4.63% drop for those who reported they had taught multiple representations at the eighth-grade level. The reduced percentage of the students' teachers who reported teaching these algebra topics at the eighth-grade level suggests that some algebra topics may have been introduced at earlier grades in the later years. The Ministry of Education in Botswana requires teachers to introduce algebra to students through a gradual shift from using numbers to using letters between the fifth to seventh grades. Also, students are expected to be able to simplify linear equations and solve algebraic equations using trial and error and substitution methods at the seventh-grade level (Kesamang, Mudongo, Keatimilwe & Botaane, 2012). However, from the teachers' reports, there is still a significant proportion of teachers who had not taught the content before eighth grade.

After the seventh grade, students take a three-year junior certificate course during which it is expected they expand their algebra knowledge to include Gaussian elimination and substitution to solve simultaneous equations with two unknowns (Kesamang, Mudongo, Keatimilwe & Botaane, 2012). At the end of the three-year period, the students should be able to expand and factor linear equations. The findings from this study indicate that more than 50% of the teachers were meeting the requirement of the ministry of education by focusing more on the required algebraic topics by the time the students had completed their eighth grade. However, it is worth noting that a considerable proportion of the teachers in Botswana had not taught the students' *simultaneous equation*s and *multiple representations* of functions by the end of eighth-grade.

Singapore

Most of the algebra topics were taught in eighth grade. However, between 2011 and 2015, more teachers taught *simplifying and evaluating algebraic expressions, representations of functions,* and *simple linear equations and inequalities* before the eighth grade. Notably, more than 60% of the teachers reported they taught *numeric, algebraic and geometric patterns* before eighth grade. The expectations from the Ministry of Education in Singapore stress that more algebra should be taught in the seventh and eighth grade. Between the seventh and eighth grade students are expected to be able to manipulate algebraic expressions and formulae of linear and quadratic expressions. Also, students should have known linear and quadratic functions and simultaneous equations at these levels. Further, the teachers are expected to teach linear inequalities with one unknown (Chin et al., 2012). However, between first and sixth grade the students should be taught "algebraic expressions in one variable" (Chin et al., 2012, p. 807). From the findings in this study, it seems that more of the students' teachers taught content per the expectations of the Ministry of Education in the algebra topics they taught before and at eighth grade.

The United States

Most of the algebra was taught at the eighth-grade level. However, before eighth grade, there was a significant drop in the percentage of teachers who reported they taught *numeric algebraic and geometric patterns and sequences* before eighth grade between 2011 and 2015. In contrast, there was a significant increase in the percentage of teachers who reported they taught *simplifying and evaluating expressions* before eighth grade between 2011 and 2015. Also, there was a 5% increase in the percentage of teachers who taught simple linear equations and inequalities before eighth grade. Over the two TIMSS cycles, more than 75% of teachers reported they taught *properties of functions* and *representations of functions* at the eighth-grade level. Sen, Malley, Hodson, and Werwath (2012) documented that two of the three critical focal areas in algebra for eighth grade include, "formulating and reasoning about expressions and equations, solving linear equation…function concept and using functions to describe quantitative relationships" (p.983). Further, learning *patterns* in algebra were expected between the third and fifth grade, and in the seventh grade (NGA & CCSSO, 2010). Although some states have adopted the CCSSM since 2010, it seems that more of the students' teachers taught *numeric, algebraic and geometric patterns* at the eighth-grade level even though the intended curriculum indicated from fourth grade onwards.

In sum, in the three countries, the teachers' reports indicate a possibility of a shift in the emphasis of particular algebra topics across the grades over the years between 2011 and 2015. In particular, there seemed to be more emphasis on teaching *numeric, algebraic and geometric patterns* in Singapore and Botswana before eighth grade. The difference in algebra achievement may be attributed to the shift in the emphasis of the various topics in the grade levels. A discussion of the relationships between OTL algebra topics and the eighth-grade students' algebra achievement would provide more information on the influence of the OTL.

Relationships Between OTL Algebra and Algebra Achievement

The findings from the multilevel model of the relationships between OTL algebra and eighth-grade students' algebra achievement suggest that the grade level for teaching algebra topics mattered. Specifically, the students whose teachers reported having taught them *simple linear equations and inequalities* and *simultaneous equations* in the Singapore before eighth grade had significantly higher scores than those who had not been taught these topics or had these OTL at a later grade. Similarly, in the United States students who had been taught *properties of functions* and *simplifying and evaluating algebraic expressions* before eighth grade, had significantly higher scores.

Particular topics taught at the eighth-grade level could have positioned some students at an advantage in their algebra achievement. For instance, eighth-grade students whose teachers had taught (i) *numeric, algebraic, and geometric patterns (ii)simplifying and evaluating algebraic expressions,* and *(iii) simultaneous equations* at eighth grade in the United States, performed significantly higher. In contrast, the eighth-grade students who had been taught *simple linear equations and inequalities* at eighth grade in Singapore had lower scores, whereas those who were taught *simultaneous equations* at the same level had significantly higher algebra scores. These findings support the notion that, learning some algebra topics in the early grades could be more advantageous for students, such as the Singaporean case for learning *simple linear equations and inequalities*.

The higher scores in algebra achievement in Singapore and the United States could be attributed to the approaches used to teach the algebra topics and the systemic policies allowing for teaching the topics earlier. For example, numeric, algebraic, and geometric patterns were to be taught from the fifth through eighth grade in the United States and from the first through eighth grade in Singapore. However, the intended algebra curriculum indicated that topics that incorporate patterns were to be taught from the third grade through eighth grade. Chan (2015) reported using manipulatives such as pattern blocks increased the understanding of algebraic patterns in early grades. A sequential development using representations from concrete to visual and then to symbolic supported students in identifying patterns and making generalizations (Chan, 2015). Although the data did not indicate the approaches used to teach algebra, it is probable that there could have been particular approaches used to introduce this topic in Singapore and the United States that were different than those used to teach the students in Botswana. Perhaps these approaches to teaching could have made the difference in students understanding of the algebra topic.

These findings support previous findings by Blanton and Kaput (2004) in which they reported that students in early grades could understand covariation and functional thinking in algebra. From the multi-level model, the findings show that in the United States, students whose teachers reported they had taught properties of functions before eighth-grade had significantly higher scores. Although the descriptive findings showed that only 6% of the students' teachers in the United States reported teaching this topic before eighth-grade, their students had significantly higher algebra scores. Gardiner and Sawrey (2016) found that with the intentional selection of tasks, the promotion of functional thinking is possible amongst first graders. Likewise, the teacher moves that foster persistence through effective questioning were found to be effective in teaching the discovery of patterns and the relationships between variables (Whitin & Whitin, 2014). In sum, the findings show that functional thinking in the early grades can be of advantage for students' success in algebraic thinking.

Implications for Practice and Research

These findings indicate that students enter high school with different algebra competencies within countries. Notably, some of these gaps in their algebra knowledge may or may not be filled during high school. In Botswana, students are taught less algebra in the elementary years than in Singapore and the United States. As indicated in this study, some of the algebra topics can be taught at the lower grades. Perhaps using particular instructional practices and problem-solving strategies make this possible. In the literature (e.g. Gardiner & Sawrey,2016 ; Whitin & Whitin, 2014), algebraic thinking can be promoted in lower elementary school grades. Perhaps what would be beneficial is promoting approaches to teaching algebraic thinking amongst pre-service and in-service teachers. In particular teaching *simple linear equations and inequalities, topics on patterns*, *numeric, algebraic and geometric patterns, simplifying and evaluating algebraic expressions,* and *functional reasoning* are some topics that should be discussed in pedagogy-related professional development forums. Most important, is selecting cognitively demanding tasks (Smith & Stein, 2011) that allow for students to develop conceptual understanding of these algebra topics and developing strategies that promote algebraic thinking.

Further studies that examine how secondary teachers fill algebra knowledge gaps in high school mathematics should be investigated. Also, studies of the OTL algebra based on student backgrounds might provide insights on which students are not learning the early algebra and the effects of these gaps on their continual success advanced mathematics. The findings point to the need for further research on the instructional strategies used in the Singapore countries for teaching *simultaneous equations* and *simple linear equations and inequalities* in before eighth grade that were related to a significant higher algebraic score. Also, the pedagogy related to the successful learning of *functional representation* and *simplifying and evaluating algebraic expressions* before eighth grade in the United States is an agenda for research.

Therefore, OTL various algebra topics points out an important factor that should be considered in educational policy. For example, there is a need for more discussions and research on (a) important topics to be learned in algebra; (b) investigations on the coherence of the algebra curriculum at the elementary levels in the schools across different countries; and (c) the time needed to teach algebra in early grades and middle school. Finally, the need for professional development on approaches to teaching algebra in the earlier grades using research-based methods of instruction.

Notes

1. SPSS is a software package used for statistical analysis that works on windows and macintosh operating systems. It is presently produced by IBM.
2. HLM software is used for analysis of a hierarchical linear model. These models provide improved estimates for within and between groups, cross level effects and variance and covariance components.

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